Analysis of Multicell Cooperation with Random User Locations Via Deterministic Equivalents

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***Abstract*—We consider the uplink of a one-dimensional 2-cell network with fixed base stations (BSs) and randomly distributed user terminals (UTs). Assuming that the number of antennas per BS and the number of UTs grow infinitely large, we derive tight approximations of the ergodic sum rate with and without multicell processing for optimal and sub-optimal detectors. We use these results to find the optimal BS placement to maximize the system capacity. This work can be seen as a first attempt to apply large random matrix theory to the study of networks with random topologies. We demonstrate that such an approach is feasible and leads to analytically tractable expressions of the average system performance. Moreover, these results can be used to optimize certain system parameters for a given distribution of user terminals and to assess the gains of multicell cooperation.**

I. INTRODUCTION

Multicell processing or base station (BS) cooperation is an effective means to counter intercell interference and to increase the spectral efficiency of mobile networks [1], [2], [3]. Although this topic is under heavy research since several years, the theoretical analysis has been limited for a long time to simple Wyner-type models [4], [5] or simulations [6]. Only recently, more complex system models accounting for realistic features, such as limited backhaul capacity, imperfect channel state information (CSI), and path loss, were considered using asymptotic results of large random matrix theory (RMT) [7], [8], [9], [10]. However, all these works assume a *deterministic placement* of the user terminals (UTs) and BSs, and RMT is only used to average over the random channel gains.

A promising approach to deal with large *random* networks is stochastic geometry [11], [12]. This technique assumes that the UTs and the BSs form independent spatial point processes with known stochastic properties. The main goal is then to characterize the distribution of the signal-to-interference-plus-noise ratio (SINR) at a typical receiver and to derive related metrics such as the throughput or the outage probability. Although a very powerful tool, stochastic geometry has not yet led to tractable results considering multicell processing. First steps in this direction were taken in [13], [14]. However, these works consider only interference coordination but not data sharing or joint decoding/precoding.

The aim of this paper is to extend the existing random matrix methods for the analysis of multicell cooperative systems to account for random user locations. This allows one to find approximations of the average system performance (with respect to fading and to user locations) and to answer questions of the type: For a given area and user distribution, where should one deploy the BSs? How much do we gain on average from multicell cooperation? How does cooperation affect the optimal BS placement? In order to explain our approach and to keep the presentation simple, we restrict ourselves in this work to a one-dimensional network consisting of two BSs and randomly deployed UTs on a line. Under a large system limit where the number of antennas per BSs and the the number of UTs grow infinitely large, we derive tight approximations of the uplink sum-rate with and without multicell processing for optimal and sub-optimal detectors. We then leverage these results to find the BS placement which maximizes the system capacity. Simulations demonstrate that the asymptotic results provide tight performance approximations for realistic system dimensions. Many extensions of this work are possible, e.g., 2/3-dimensional networks topologies, downlink transmissions, or more realistic path-loss models accounting for directional antennas.

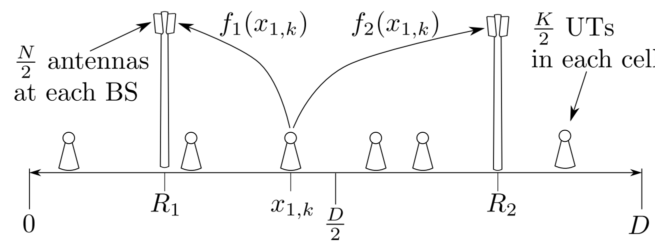

Fig. 1. Sketch of the system model.

II. SYSTEM MODEL

The focus of this paper is on the uplink of a one-dimensional system where two BSs and K UTs are located on a line of length D (see Fig. 1). We assume that one BS and half of the UTs are located in each of the intervals $[0, D/2]$ and $[D/2, D]$. These will be referred to as cell 1 and 2, respectively. Each UT is indexed by a couple (i, k), $i \in \{1, 2\}$, $k \in \{1, \dots, K/2\}$. The BSs are equipped with $N/2$ antennas; the UTs have a single antenna. BS i is located at position R_i; UT k in cell i is located at position $x_{i,k}$. The UTs in both cells are assumed to be randomly uniformly distributed. We will distinguish between two scenarios: cooperation and no cooperation. In the first scenario, both BSs jointly decode the messages for the UTs in both cells. We ignore any practical constraints, such as limited backhaul capacity, and assume that the BSs can cooperate without any restriction. Thus, they can be seen as a distributed antenna system with N antennas. In the second scenario, each BS only decodes the messages from the UTs in its own cell.

A. Uplink channel model

Denote $\mathbf{y}_i \in \mathbb{C}^{N/2}$ the received signal vector a BS i. Then, the stacked received signal vector $\mathbf{y} \in \mathbb{C}^N$ at both BSs reads

$$\mathbf{y} = \begin{pmatrix}\mathbf{y}_1\\ \mathbf{y}_2\end{pmatrix} = \sqrt{\rho_{\text{ul}}}\mathbf{H}\mathbf{s} + \mathbf{n} = \sqrt{\rho_{\text{ul}}}\begin{pmatrix}\mathbf{H}_1\\ \mathbf{H}_2\end{pmatrix}\begin{pmatrix}\mathbf{s}_1\\ \mathbf{s}_2\end{pmatrix} + \begin{pmatrix}\mathbf{n}_1\\ \mathbf{n}_2\end{pmatrix}$$
$$= \sqrt{\rho_{\text{ul}}}\begin{pmatrix}\mathbf{H}_{1,1} & \mathbf{H}_{1,2}\\ \mathbf{H}_{2,1} & \mathbf{H}_{2,2}\end{pmatrix}\begin{pmatrix}\mathbf{s}_1\\ \mathbf{s}_2\end{pmatrix} + \begin{pmatrix}\mathbf{n}_1\\ \mathbf{n}_2\end{pmatrix} \quad (1)$$

where $\mathbf{s}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{K/2})$ is the transmit vector of the UTs in cell i, ρ_{ul} is the transmit signal-to-noise ratio (SNR), $\mathbf{n}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N/2})$ is a noise vector at BS i, and $\mathbf{H}_{i,j} = \left[\mathbf{h}_{i,j,1} \cdots \mathbf{h}_{i,j,K/2}\right] \in \mathbb{C}^{N/2\times K/2}$ is the channel matrix from the UTs in cell j to BS i. We model $\mathbf{H}_{i,j}$ as

$$\mathbf{H}_{i,j} = \frac{1}{\sqrt{K}}\mathbf{G}_{i,j}\mathbf{T}_{i,j}^{\frac{1}{2}} \quad (2)$$

where $\mathbf{G}_{i,j} \in \mathbb{C}^{N/2\times K/2}$ is a standard complex Gaussian matrix, $\mathbf{T}_{i,j} = \text{diag}(f_i(x_{j,k}))_{k=1}^{K/2} \in (\mathbb{R}^+)^{K/2\times K/2}$ is a path loss matrix, with $f_i(x)$ the path loss function from a UT at position x to BS i. We assume the widely used path loss model

$$f_i(x) = \frac{1}{(1+|R_i - x|)^{\beta}} \quad (3)$$

where β is a path loss exponent and the term "1+" ensures that f_i is bounded. However, any other bounded path loss function could be considered. One could for example account for different heights of the BSs or introduce more complex path loss functions, e.g., to model directional antennas.

B. Performance measures

We consider two different performance measures, namely the ergodic mutual information and the ergodic sum-rate with minimum-mean-square-error (MMSE) detection, normalized by N. Both quantities must be understood as averages over both the channel realizations and the positions of the UTs. We assume that full CSI is available at the BSs, while the UTs are unaware of the channel realizations.

1) Cooperation: For the case of full cooperation, the ergodic mutual information and MMSE sum-rate per antenna are respectively defined as ($\log(x)$ denotes the natural logarithm)

$$I_{\text{c}}(\rho_{\text{ul}}) = \frac{1}{N}\mathbb{E}\left[\log\det\left(\mathbf{I}_N + \rho_{\text{ul}}\mathbf{H}\mathbf{H}^{\mathsf{H}}\right)\right] \quad (4)$$

and

$$R_{\text{c}}^{\text{MMSE}}(\rho_{\text{ul}}) = \frac{1}{N}\sum_{i=1}^{2}\sum_{k=1}^{K/2}\mathbb{E}\left[\log\left(1+\gamma_{i,k}^{\text{c}}(\rho_{\text{ul}})\right)\right] \quad (5)$$

where

$$\gamma_{i,k}^{\text{c}}(\rho_{\text{ul}}) = \mathbf{h}_{i,k}^{\mathsf{H}}\left(\mathbf{H}\mathbf{H}^{\mathsf{H}} - \mathbf{h}_{i,k}\mathbf{h}_{i,k}^{\mathsf{H}} + \frac{1}{\rho_{\text{ul}}}\mathbf{I}_N\right)^{-1}\mathbf{h}_{i,k} \quad (6)$$

and $\mathbf{h}_{i,k} \in \mathbb{C}^N$ is the kth column of the matrix $(\mathbf{H}_{1,i}^{\mathsf{T}}\mathbf{H}_{2,i}^{\mathsf{T}})^{\mathsf{T}}$.

2) No Cooperation: For the case of no cooperation, both quantities are respectively defined as

$$I_{\text{nc}}(\rho_{\text{ul}}) = \frac{1}{N}\sum_{i=1}^{2}\mathbb{E}\left[\log\det\left(\mathbf{I}_N + \rho\mathbf{H}_i\mathbf{H}_i^{\mathsf{H}}\right)\right.$$
$$\left. - \log\det\left(\mathbf{I}_N + \rho\mathbf{H}_{i,\bar{i}}\mathbf{H}_{i,\bar{i}}^{\mathsf{H}}\right)\right] \quad (7)$$

where $\bar{i} = \text{mod}(i,2)+1$ and

$$R_{\text{nc}}^{\text{MMSE}}(\rho_{\text{ul}}) = \frac{1}{N}\sum_{i=1}^{2}\sum_{k=1}^{K/2}\mathbb{E}\left[\log\left(1+\gamma_{i,k}^{\text{nc}}(\rho_{\text{ul}})\right)\right] \quad (8)$$

where

$$\gamma_{i,k}^{\text{nc}}(\rho_{\text{ul}}) = \frac{\left|\mathbf{h}_{i,i,k}^{\mathsf{H}}\mathbf{Q}_{i,k}\mathbf{h}_{i,i,k}\right|^2}{\mathbf{h}_{i,i,k}^{\mathsf{H}}\mathbf{Q}_{i,k}\mathbf{h}_{i,i,k} + \mathbf{h}_{i,i,k}^{\mathsf{H}}\mathbf{Q}_{i,k}\mathbf{H}_{i,\bar{i}}\mathbf{H}_{i,\bar{i}}^{\mathsf{H}}\mathbf{Q}_{i,k}\mathbf{h}_{i,i,k}} \quad (9)$$

and

$$\mathbf{Q}_{i,k} = \left(\mathbf{H}_{i,i}\mathbf{H}_{i,i}^{\mathsf{H}} - \mathbf{h}_{i,i,k}\mathbf{h}_{i,i,k}^{\mathsf{H}} + \frac{1}{\rho_{\text{ul}}}\mathbf{I}_{N/2}\right)^{-1}. \quad (10)$$

III. Deterministic equivalents: From fixed to random user locations

Computing the above performance measures for finite system dimensions is in general intractable by exact analysis. We will therefore consider a large system limit where N and K grow infinitely large at the same speed. This allows us to derive asymptotically tight approximations of all quantities which are shown by simulations to be accurate for small (N,K). We will first recall some existing results of RMT (slightly adapted to our notations). These will be extended to account for random user locations and are needed for the derivations in Section IV.

Theorem 1 ([15, Theorems 2.4, 4.1 and Lemma 6.1]): Let $\mathbf{Y} \in \mathbb{C}^{N\times N'}$ be defined as

$$\mathbf{Y} = \begin{pmatrix}\mathbf{X}_{1,1}\mathbf{D}_{1,1}^{1/2} & \cdots & \mathbf{X}_{1,C}\mathbf{D}_{1,C}^{1/2}\\ \vdots & \ddots & \vdots\\ \mathbf{X}_{B,1}\mathbf{D}_{B,1}^{1/2} & \cdots & \mathbf{X}_{B,C}\mathbf{D}_{B,C}^{1/2}\end{pmatrix}$$

where $\mathbf{X}_{i,j} \in \mathbb{C}^{N_i\times n_j}$ is random with i.i.d. entries $[\mathbf{X}_{i,j}]_{k,l} \sim \mathcal{CN}(0, 1/n)$ and $\mathbf{D}_{i,j} = \text{diag}(g_i(z_{j,k}))_{k=1}^{n_j}$ for some nonnegative bounded function g_i and sequence of reals $(z_{j,k})_{1\le k\le n_j}$. Denote $N = \sum_i N_i$, $N' = \sum_j n_j$, $c_i = \frac{N_i}{n}$, and $\bar{c}_i = \frac{N_i}{N}$. Assume that $N_1,\ldots,N_B,n_1,\ldots,n_J,n \to \infty$ such that $0 < \liminf \frac{N_i}{n_j} \le \limsup \frac{N_i}{n_j} < \infty\ \forall i,j$ and $0 < \liminf c_i \le \limsup c_i < \infty\ \forall i$. Then, for any $\rho > 0$,

$$\frac{1}{N}\text{tr}\,\mathbf{R}\left(\mathbf{Y}\mathbf{Y}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-1} - \frac{1}{N}\text{tr}\,\mathbf{R}\boldsymbol{\Psi} \xrightarrow{\text{a.s}} 0$$

where $\boldsymbol{\Psi} = \text{diag}(\Psi_1\mathbf{I}_{N_1},\ldots,\Psi_B\mathbf{I}_{N_B})$ and $(\Psi_1,\ldots,\Psi_B)$ is the unique solution to the following set of B implicit equations

$$\Psi_i = \left(\frac{1}{\rho} + \frac{1}{n}\sum_{j=1}^{C}\sum_{k=1}^{n_j}\frac{g_i(z_{j,k})}{1+\sum_{b=1}^{B}c_b g_b(z_{j,k})\Psi_b}\right)^{-1}$$

such that $\Psi_i \geq 0$ for $i = 1, \dots, B$. Moreover,

$$\frac{1}{N}\mathbb{E}\left[\log\det\left(\mathbf{I}_N + \rho\mathbf{Y}\mathbf{Y}^{\mathsf{H}}\right)\right] - V(\rho) \to 0$$

where

$$\begin{aligned} V(\rho) = &\sum_{i=1}^{B} \bar{c}_i \log\left(\frac{\rho}{\Psi_i}\right) \\ &+ \frac{1}{N}\sum_{j=1}^{C}\sum_{k=1}^{n_j} \log\left(1 + \sum_{i=1}^{B} c_i g_i(z_{j,k})\Psi_i\right) \\ &- \frac{1}{N}\sum_{j=1}^{C}\sum_{k=1}^{n_j} \frac{\sum_{i=1}^{B} c_i g_i(z_{j,k})\Psi_i}{1+\sum_{i=1}^{B} c_i g_i(z_{j,k})\Psi_i}. \end{aligned}$$

Theorem 2: Under the assumptions of Theorem 1, the following holds

$$\frac{1}{N}\operatorname{tr}\mathbf{R}\left(\mathbf{Y}\mathbf{Y}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-2} - \frac{1}{N}\operatorname{tr}\mathbf{R}\mathbf{\Psi}' \xrightarrow{\text{a.s.}} 0$$

where $\mathbf{\Psi}' = \operatorname{diag}(\Psi_1'\mathbf{I}_{N_1}, \dots, \Psi_B'\mathbf{I}_{N_B})$ and $\underline{\mathbf{\Psi}}' = [\Psi_1' \cdots \Psi_B']^{\mathsf{T}}$ is given as

$$\underline{\mathbf{\Psi}}' = \left(\mathbf{I}_B - \mathbf{J}\right)^{-1}\mathbf{v}$$

for $\mathbf{v} = [\Psi_1^2 \cdots \Psi_B^2]^{\mathsf{T}}$ and $\mathbf{J} \in (\mathbb{R}^+)^{B\times B}$ with elements

$$[\mathbf{J}]_{i,b} = \frac{1}{n}\sum_{j=1}^{C}\sum_{k=1}^{n_j}\frac{c_b g_b(z_{j,k}) g_i(z_{j,k})\Psi_i^2}{\left(1+\sum_{l=1}^{B} c_l g_l(z_{j,k})\Psi_l\right)^2}$$

where $(\Psi_1, \dots, \Psi_B)$ is given by Theorem 1.

In the above theorems, the functions g_i can be identified with our path loss functions f_i and the quantities $z_{j,k}$ with the positions of the UTs $x_{i,k}$. However, the values of $z_{j,k}$ are assumed to be deterministic while we require them to be random. The following propositions extend Theorems 1 and 2, respectively, to the case where $z_{j,1}, \dots, z_{j,n_j}$ are i.i.d. random variables with distribution F_j.

Proposition 1: Under the conditions of Theorem 1, assume additionally that $(z_{j,k})_{1\leq k\leq n_j}$ is a sequence of i.i.d. random variables with distribution F_j, for all j. Then,

$$\frac{1}{N}\operatorname{tr}\mathbf{R}\left(\mathbf{Y}\mathbf{Y}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-1} - \frac{1}{N}\operatorname{tr}\mathbf{R}\bar{\mathbf{\Psi}} \xrightarrow{\text{a.s}} 0$$

where $\bar{\mathbf{\Psi}} = \operatorname{diag}(\bar{\Psi}_1\mathbf{I}_{N_1}, \dots, \bar{\Psi}_B\mathbf{I}_{N_B})$ and $(\bar{\Psi}_1, \dots, \bar{\Psi}_B)$ is the unique solution to the following set of B implicit equations

$$\bar{\Psi}_i = \left(\frac{1}{\rho} + \sum_{j=1}^{C}\frac{n_j}{n}\int\frac{g_i(z)}{1+\sum_{b=1}^{B} c_b g_b(z)\bar{\Psi}_b}dF_j(z)\right)^{-1} \tag{11}$$

such that $\bar{\Psi}_i \geq 0$ for $i = 1, \dots, B$. Moreover,

$$\frac{1}{N}\mathbb{E}\left[\log\det\left(\mathbf{I}_N + \rho\mathbf{Y}\mathbf{Y}^{\mathsf{H}}\right)\right] - \bar{V}(\rho) \to 0$$

where

$$\begin{aligned} \bar{V}(\rho) = &\sum_{i=1}^{B} \bar{c}_i \log\left(\frac{\rho}{\bar{\Psi}_i}\right) \\ &+ \sum_{j=1}^{C}\frac{n_j}{N}\int \log\left(1 + \sum_{i=1}^{B} c_i g_i(z)\bar{\Psi}_i\right)dF_j(z) \\ &- \sum_{j=1}^{C}\frac{n_j}{N}\int \frac{\sum_{i=1}^{B} c_i g_i(z)\bar{\Psi}_i}{1+\sum_{i=1}^{B} c_i g_i(z)\bar{\Psi}_i}dF_j(z). \end{aligned}$$

Sketch: The uniqueness of solutions to the fixed-point equations (11) can be easily proved by arguments from standard interference functions [16] (see, e.g., [17] for a detailed explanation of this approach). Under the assumption that $z_{j,1}, \dots, z_{j,n_j}$ are i.i.d. with distribution F_j, it follows from the strong law of large numbers (SLLN) and the boundedness of the functions g_i that

$$\begin{aligned} &\frac{1}{n_j}\sum_{k=1}^{n_j}\frac{g_i(z_{j,k})}{1+\sum_{b=1}^{B} c_b g_b(z_{j,k})\Psi_b} \\ &\xrightarrow[n_j\to\infty]{\text{a.s.}} \int\frac{g_i(z)}{1+\sum_{b=1}^{B} c_b g_b(z)\Psi_b}dF_j(z). \end{aligned} \tag{12}$$

Using this observation, one can show that for some constant A and for all $0 < \rho < \sqrt{A^{-1}}$, the following holds:

$$\max_i |\Psi_i - \bar{\Psi}_i| \leq \frac{\epsilon_n}{1 - A\rho^2} \tag{13}$$

where $\epsilon_n \xrightarrow{\text{a.s.}} 0$ (see, e.g., [17, Theorem 4] for a similar proof). By the Vitali convergence theorem [18], it follows that the analytic extension of $\Psi_i - \bar{\Psi}_i$ to $\mathbb{C}$ is an analytic function and that the following convergence holds for *all* $\rho > 0$:

$$\Psi_i - \bar{\Psi}_i \xrightarrow{\text{a.s.}} 0 \qquad \forall i. \tag{14}$$

The last step is then to show that

$$\frac{1}{N}\mathbb{E}\left[\log\det\left(\mathbf{I}_N + \rho\mathbf{Y}\mathbf{Y}^{\mathsf{H}}\right)\right] - \bar{V}(\rho) \to 0. \tag{15}$$

This can be done by dominated convergence arguments similar to the proof of [15, Theorem 4.1]. ■

Proposition 2: Under the conditions of Theorem 1, assume additionally that $(z_{j,k})_{1\leq k\leq n_j}$ is a sequence of i.i.d. random variables with distribution F_j, for all j. Then,

$$\frac{1}{N}\operatorname{tr}\mathbf{R}\left(\mathbf{Y}\mathbf{Y}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-2} - \frac{1}{N}\operatorname{tr}\mathbf{R}\bar{\mathbf{\Psi}}' \xrightarrow{\text{a.s.}} 0$$

where $\bar{\mathbf{\Psi}}' = \operatorname{diag}(\bar{\Psi}_1'\mathbf{I}_{N_1}, \dots, \bar{\Psi}_B'\mathbf{I}_{N_B})$ and $\underline{\bar{\mathbf{\Psi}}}' = [\bar{\Psi}_1' \cdots \bar{\Psi}_B']^{\mathsf{T}}$ is given as

$$\underline{\bar{\mathbf{\Psi}}}' = \left(\mathbf{I}_B - \bar{\mathbf{J}}\right)^{-1}\bar{\mathbf{v}}$$

for $\bar{\mathbf{v}} = [\bar{\Psi}_1^2 \cdots \bar{\Psi}_B^2]^{\mathsf{T}}$ and $\bar{\mathbf{J}} \in (\mathbb{R}^+)^{B\times B}$ with elements

$$[\bar{\mathbf{J}}]_{i,b} = \sum_{j=1}^{C}\frac{n_j}{n}\int\frac{c_b g_b(z) g_i(z)\bar{\Psi}_i^2}{\left(1+\sum_{l=1}^{B} c_l g_l(z)\bar{\Psi}_l\right)^2}dF_j(z)$$

where $(\bar{\Psi}_1, \dots, \bar{\Psi}_B)$ is given by Proposition 1.

Proof: First, by the SLLN

$$\frac{1}{n}\sum_{j=1}^{C}\sum_{k=1}^{n_j}\frac{c_b g_b(z_{j,k})g_i(z_{j,k})\Psi_i^2}{\left(1+\sum_{l=1}^{B}c_l g_l(z_{j,k})\Psi_l\right)^2} - \sum_{j=1}^{C}\frac{n_j}{n}\int\frac{c_b g_b(z)g_i(z)\Psi_i^2}{\left(1+\sum_{l=1}^{B}c_l g_l(z)\Psi_l\right)^2}dF_j(z)\xrightarrow{\text{a.s.}}0 \quad (16)$$

where Ψ_i are defined in Theorem 1. Second, $\Psi_i-\bar{\Psi}_i\xrightarrow{\text{a.s.}}0\ \forall i$, as shown in the proof of Proposition 1. Thus,

$$[\mathbf{J}]_{i,b}-\left[\bar{\mathbf{J}}\right]_{i,b}\xrightarrow{\text{a.s.}}0\ \forall i,b \quad (17)$$

where $\mathbf{J}$ is given by Theorem 2. This implies that

$$\Psi_i'-\bar{\Psi}_i'\xrightarrow{\text{a.s.}}0\ \forall i. \quad (18)$$

■

IV. Main results

Equipped with the results from the last section, we are now able to derive large system approximations of the performance measures introduced in Section II. We assume from now on that $N,K\to\infty$ while $0<\liminf c\leq\limsup c<\infty$, where $c=\frac{N}{K}$. Since the UTs in both cells are randomly uniformly distributed over the intervals $[0,D/2]$ and $[D/2,D]$, respectively, it follows that $x_{i,k}\sim F_i$, where F_i has density

$$dF_i(x)=\begin{cases}\frac{2}{D}\mathbb{1}\left(0\leq x\leq\frac{D}{2}\right) & ,\ i=1\\ \frac{2}{D}\mathbb{1}\left(\frac{D}{2}\leq x\leq D\right) & ,\ i=2\end{cases}. \quad (19)$$

The application of Proposition 1 leads then to our first result:

Proposition 3 (Mutual information with cooperation):

$$I_{\text{c}}(\rho_{\text{ul}})-\bar{I}_{\text{c}}(\rho_{\text{ul}})\xrightarrow[N\to\infty]{}0$$

where

$$\bar{I}_{\text{c}}(\rho_{\text{ul}})=\frac{1}{2}\sum_{i=1}^{2}\log\left(\frac{\rho_{\text{ul}}}{\psi_i}\right)+\frac{1}{cD}\int_0^D\log\left(1+\frac{c}{2}\sum_{i=1}^{2}f_i(x)\psi_i\right)dx-\frac{1}{cD}\int_0^D\frac{\frac{c}{2}\sum_{i=1}^{2}f_i(x)\psi_i}{1+\frac{c}{2}\sum_{i=1}^{2}f_i(x)\psi_i}dx$$

and $(\psi_1,\psi_2)\in(\mathbb{R}^+)^2$ are given as the unique fixed-point of

$$\psi_i=\left(\frac{1}{\rho_{\text{ul}}}+\frac{1}{D}\int_0^D\frac{f_i(x)}{1+\frac{c}{2}\sum_{b=1}^{2}f_b(x)\psi_b}dx\right)^{-1},\ i=1,2.$$

Our next result provides an asymptotically tight approximation of the sum-rate with MMSE detection:

Proposition 4 (MMSE Sum-rate with cooperation):

$$R_{\text{c}}^{\text{MMSE}}(\rho_{\text{ul}})-\bar{R}_{\text{c}}^{\text{MMSE}}(\rho_{\text{ul}})\xrightarrow[N\to\infty]{}0$$

where

$$\bar{R}_{\text{c}}^{\text{MMSE}}(\rho_{\text{ul}})=\frac{1}{cD}\int_0^D\log\left(1+\frac{c}{2}\sum_{i=1}^{2}f_i(x)\psi_i\right)dx$$

and (ψ_1,ψ_2) are defined in Proposition 3.

Proof: By Lemmas 1 and 2, the following holds

$$\frac{1}{K}\text{tr}\begin{pmatrix}f_1(x_{i,k})\mathbf{I}_{N/2} & \mathbf{0}\\ \mathbf{0} & f_2(x_{i,k})\mathbf{I}_{N/2}\end{pmatrix}\left(\mathbf{H}\mathbf{H}^{\mathsf{H}}+\frac{1}{\rho_{\text{ul}}}\mathbf{I}_N\right)^{-1}-\gamma_{i,k}^{\text{c}}\xrightarrow{\text{a.s.}}0. \quad (20)$$

Direct application of Proposition 1 to the first term leads to

$$\gamma_{i,k}^{\text{c}}-\frac{c}{2}\sum_{b=1}^{B}f_b(x_{i,k})\psi_i\xrightarrow{\text{a.s.}}0. \quad (21)$$

By the SLLN and (21), we have

$$\frac{1}{N}\sum_{i=1}^{2}\sum_{k=1}^{K/2}\log\left(1+\gamma_{i,k}^{\text{c}}\right)-\frac{1}{cD}\int_0^D\log\left(1+\frac{c}{2}\sum_{b=1}^{B}f_b(x_{i,k})\psi_i\right)\xrightarrow{\text{a.s.}}0. \quad (22)$$

Since the functions f_i are bounded and $\psi_i\leq\rho$, it follows from dominated convergence arguments that the last convergence also holds in the first mean. This concludes the proof. ■

Next, we provide a deterministic equivalent of the normalized ergodic mutual information without cooperation:

Proposition 5 (Mutual information without cooperation):

$$I_{\text{nc}}(\rho_{\text{ul}})-\bar{I}_{\text{nc}}(\rho_{\text{ul}})\xrightarrow[N\to\infty]{}0$$

where

$$\bar{I}_{\text{nc}}(\rho_{\text{ul}})=\sum_{i=1}^{2}\bar{I}_{i,i}(\rho_{\text{ul}})-\bar{I}_{i,\bar{i}}(\rho_{\text{ul}})$$

with

$$\bar{I}_{i,i}(\rho_{\text{ul}})=\frac{1}{2}\log\left(\frac{\rho_{\text{ul}}}{\psi_i}\right)+\frac{1}{cD}\int_0^D\log\left(1+\frac{c}{2}f_i(x)\psi_i\right)dx-\frac{1}{cD}\int_0^D\frac{\frac{c}{2}f_i(x)\psi_i}{1+\frac{c}{2}f_i(x)\psi_i}dx$$

$$\bar{I}_{i,\bar{i}}(\rho_{\text{ul}})=\frac{1}{2}\log\left(\frac{\rho_{\text{ul}}}{2\upsilon_i}\right)+\frac{1}{2c}\int\log\left(1+cf_i(x)\upsilon_i\right)dF_{\bar{i}}(x)-\frac{1}{2c}\int\frac{cf_i(x)\upsilon_i}{1+cf_i(x)\upsilon_i}dF_{\bar{i}}(x)$$

and where $(\Psi_1,\Psi_2)\in(\mathbb{R}^+)^2$ and $(\upsilon_1,\upsilon_2)\in(\mathbb{R}^+)^2$ are given respectively as the unique fixed points of the following sets of equations:

$$\psi_i=\left(\frac{1}{\rho}+\frac{1}{D}\int_0^D\frac{f_i(x)}{1+\frac{c}{2}f_i(x)\psi_i}dx\right)^{-1},\quad i=1,2$$

$$\upsilon_i=\left(\frac{2}{\rho}+\int\frac{f_i(x)}{1+cf_i(x)\upsilon_i}dF_{\bar{i}}(x)\right)^{-1},\quad i=1,2.$$

Proof: The proof follows directly from an application of Proposition 1 to each of the four individual terms in the expression of $I_{\text{nc}}(\rho_{\text{ul}})$. ■

Our last result is an asymptotically tight approximation of the MMSE sum-rate without BS-cooperation:

Proposition 6 (MMSE Sum-rate without cooperation):

$$R_{\text{nc}}^{\text{MMSE}}(\rho_{\text{ul}}) - \bar{R}_{\text{nc}}^{\text{MMSE}}(\rho_{\text{ul}}) \xrightarrow[N\to\infty]{} 0$$

where

$$\bar{R}_{\text{nc}}^{\text{MMSE}}(\rho_{\text{ul}}) = \sum_{i=1}^{2} \bar{R}_i^{\text{MMSE}}(\rho_{\text{ul}})$$

with

$$\bar{R}_i^{\text{MMSE}}(\rho_{\text{ul}}) = \frac{1}{2c}\int \log\left(1 + \frac{c f_i(x)\upsilon_i}{1+\frac{\upsilon_i'}{2\upsilon_i}\int f_i(x)dF_{\bar{i}}(x)}\right) dF_i(x)$$

and where $(\upsilon_1, \upsilon_2) \in (\mathbb{R}^+)^2$ is the unique fixed point of the following set of equations

$$\upsilon_i = \left(\frac{2}{\rho} + \int \frac{f_i(x)}{1+cf_i(x)\upsilon_i} dF_i(x)\right)^{-1}, \quad i = 1,2$$

and $(\upsilon_1', \upsilon_2')$ are given by

$$\upsilon_i' = \frac{2\upsilon_i^2}{1 - \int \frac{cf_i^2(x)\upsilon_i^2}{(1+cf_i(x)\upsilon_i)^2} dF_i(x)}, \quad i = 1,2.$$

Sketch: By repeated application of Lemmas 1, 2, and the SLLN, one can show the following convergence:

$$\frac{cf_i(x_{i,k})\frac{1}{N}\text{tr}\left(\mathbf{H}_{i,i}\mathbf{H}_{i,i}^{\text{H}} + \frac{1}{\rho_{\text{ul}}\mathbf{I}_{N/2}}\right)^{-1}}{1 + \frac{\frac{1}{N}\text{tr}\left(\mathbf{H}_{i,i}\mathbf{H}_{i,i}^{\text{H}} + \frac{1}{\rho_{\text{ul}}\mathbf{I}_{N/2}}\right)^{-2}}{\frac{1}{N}\text{tr}\left(\mathbf{H}_{i,i}\mathbf{H}_{i,i}^{\text{H}} + \frac{1}{\rho_{\text{ul}}\mathbf{I}_{N/2}}\right)^{-1}}\frac{1}{2}\int f_i(x)dF_{\bar{i}}(x)} - \gamma_{i,k}^{\text{nc}}(\rho_{\text{ul}}) \xrightarrow[N\to\infty]{\text{a.s.}} 0. \tag{23}$$

By Proposition 1, we have

$$\frac{1}{N}\text{tr}\left(\mathbf{H}_{i,i}\mathbf{H}_{i,i}^{\text{H}} + \frac{1}{\rho_{\text{ul}}\mathbf{I}_{N/2}}\right)^{-1} - \upsilon_i \xrightarrow[N\to\infty]{\text{a.s.}} 0. \tag{24}$$

By Proposition 2, we have

$$\frac{1}{N}\text{tr}\left(\mathbf{H}_{i,i}\mathbf{H}_{i,i}^{\text{H}} + \frac{1}{\rho_{\text{ul}}\mathbf{I}_{N/2}}\right)^{-2} - \upsilon_i' \xrightarrow[N\to\infty]{\text{a.s.}} 0. \tag{25}$$

By the continuous mapping theorem, it follows that

$$\gamma_{i,k}^{\text{nc}}(\rho_{\text{ul}}) - \frac{cf_i(x_{i,k})\upsilon_i}{1+\frac{\upsilon_i'}{2\upsilon_i}\int f_i(x)dF_{\bar{i}}(x)} \xrightarrow[N\to\infty]{\text{a.s.}} 0. \tag{26}$$

Using the last result, we have by the SLLN that

$$\frac{1}{n}\sum_{i=1}^{2}\sum_{k=1}^{K/2}\log\left(1+\gamma_{i,k}^{\text{nc}}(\rho_{\text{ul}})\right) - \bar{R}_{\text{nc}}^{\text{MMSE}}(\rho_{\text{ul}}) \xrightarrow[N\to\infty]{\text{a.s.}} 0. \tag{27}$$

By dominated convergence arguments one can then show that the last convergence also holds in the first mean. This concludes the proof. ■

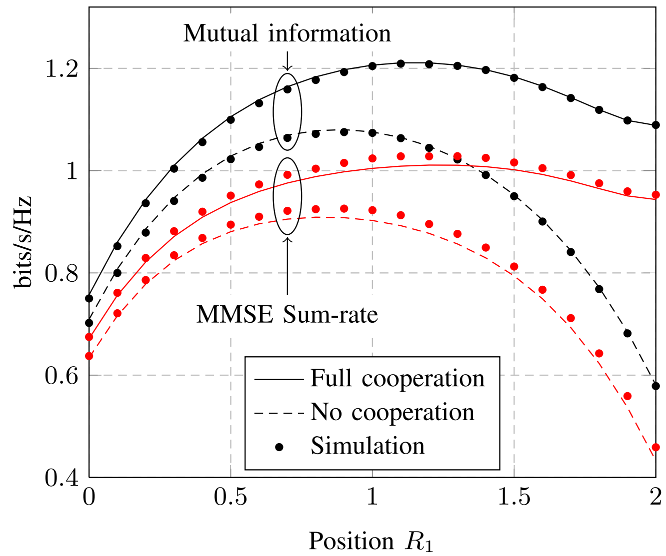

Fig. 2. I_{c}, I_{nc}, $R_{\text{c}}^{\text{MMSE}}$, and $R_{\text{nc}}^{\text{MMSE}}$ and their asymptotic approximations by Propositions 3, 4, 5, and 6 versus R_1. Markers correspond to simulation results, solid lines to the asymptotic approximations.

V. Numerical Example

Let us now verify the accuracy of the asymptotic results of the last section for a system of finite size. We assume $N = 16$ (8 antennas per BS), $K = 14$ UTs (7 UTs per cell), path loss exponent $\beta = 3.7$, transmit SNR $\rho_{\text{ul}} = 10\,\text{dB}$ and a cell radius of 1 (i.e., $D = 4$). We further suppose $R_2 = D - R_1$ so that the BSs are placed symmetrically to the inner cell edge. In Fig. 2, we show the normalized ergodic mutual information and MMSE sum-rate with and without cooperation and their asymptotic approximations by Propositions 3, 4, 5, and 6 versus R_1. We can see a good fit between the simulations and the asymptotic results over the full range of R_1; the accuracy is slightly worse for the MMSE sum-rate. Moreover, one can observe that the BSs should be located closer to the inner cell edge if they cooperate. Otherwise they should be placed closer to the outer cell edges to reduce intercell interference.

Next, we will use the asymptotic results to approximately solve an optimization problem which would have required otherwise a huge computational effort by Monte Carlo simulations. We will vary the path loss exponent β and seek to find for each value the optimal BS position R_1 which maximizes the mutual information and MMSE sum-rate with and without cooperation. In Fig. 3 and 4, we show respectively the optimal values of R_1 and the ergodic rates as a function of β. From Fig. 3, we can see that, irrespective of the type of detection and cooperation, the BSs should be located closer to their cell centers when the path loss is high. The gains of multicell processing in this regime are low as can be seen from Fig. 4. We can also observe that cooperation has a higher impact on the optimal BS placement when MMSE detection is applied. With cooperation, the BSs should be placed closer to each other than with optimal detection; without cooperation the contrary is true.

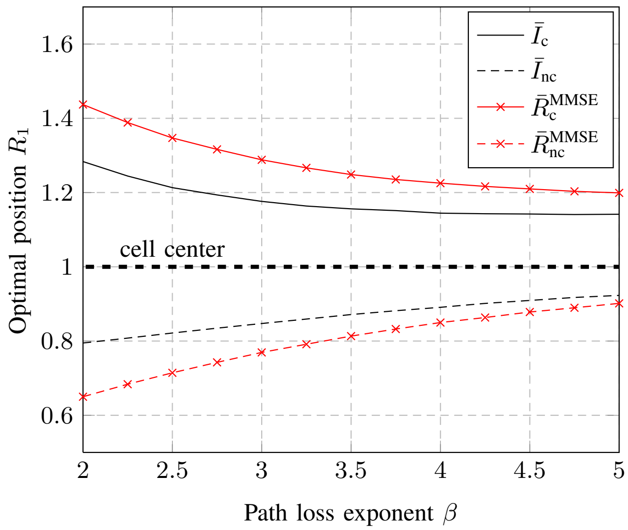

Fig. 3. Optimal BS position R_1 versus β.

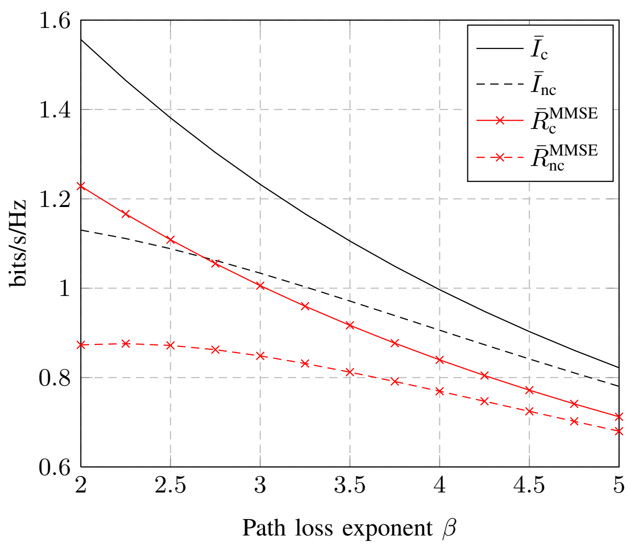

Fig. 4. Ergodic rates with optimal BS placement versus β.

VI. Conclusions

Based on RMT, we have developed deterministic equivalents for the performance analysis of cooperative multicell systems with random user locations. These results provide tight approximations of the average system performance and can be used to optimize certain system parameters, e.g., the optimal placement of BSs. Many extensions of this work are possible. Especially the combination of RMT and stochastic geometry seems a promising venue for future research.

Lemma 1 ([19, Lemma 2.7]): Let $\mathbf{A} \in \mathbb{C}^{N\times N}$ and $\mathbf{x} = [x_1 \dots x_N]^\mathsf{T} \in \mathbb{C}^N$ be a random vector of i.i.d. entries, independent of $\mathbf{A}$. Assume that $\mathbb{E}[x_i] = 0$, $\mathbb{E}\left[|x_i|^2\right] = 1$, $\mathbb{E}\left[|x_i|^8\right] < \infty$, and $\limsup_N \|\mathbf{A}\| < \infty$. Then,

$$\frac{1}{N}\mathbf{x}^\mathsf{H}\mathbf{A}\mathbf{x} - \frac{1}{N}\mathrm{tr}\,\mathbf{A} \xrightarrow{\text{a.s.}} 0.$$

Lemma 2 ([20, Lemma 2.1]): Let $z < 0$, $\mathbf{A} \in \mathbb{C}^{N\times N}$ and $\mathbf{B} \in \mathbb{C}^{N\times N}$ with $\mathbf{B}$ Hermitian nonnegative definite, and $\mathbf{x} \in \mathbb{C}^N$. Then,

$$\left|\mathrm{tr}\left(\left(\mathbf{B} - z\mathbf{I}_N\right)^{-1} - \left(\mathbf{B} + \mathbf{x}\mathbf{x}^\mathsf{H} - z\mathbf{I}_N\right)^{-1}\right)\mathbf{A}\right| \leq \frac{\|\mathbf{A}\|}{|z|}.$$

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